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STUDENT SELECTION TO UNIVERSITY ADMISSION: STABLE MATCHING THEORY AND GALE-SHAPLEY ALGORITHM

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ABSTRACT: *Strategies for improving education quality include selecting a right teaching method such as blended learning or collaborative learning, but optimal advantages of long life education can only obtained if students select right university. Many students from high school can not choose suitable universities because of the strategy shortage, which does not enable students to make an appropriate college wish list. In addition, universities always want the top students with high academic achievement, leading to many universities not meeting the enrollment quota. To tackle this problem, matching theory, a mathematical framework describing mutually beneficial relationships over time, was applied in this paper. This theory pairs the elements, students, and universities and turns them into a match named Stable Matching for Student Selection (SMSS). Related to stable matching theory, the Gale-Shapley algorithm is an useful method to solve stable matching problems, contains many checking rounds that check every university in the student's list, which leads to an equal outcome for both wishes of the students and the schools. From these ways of resolving conflict, students have opportunities to enter the most suitable universities that enable them to make the best contribution to society by meeting universities' requirements and getting flying scores. They, therefore, have appropriate jobs, which is a stepping stone to contributing to society. Besides, there will be no imbalance in student numbers between universities, which maintains a steady source of capital to develop less selective schools due to lack of students and reduces the student load for top schools.*

Keywords: *Student selection, University admissions, Stable matching theory, Gale-Shapley algorithm.*

I. INTRODUCTION

University admissions are the process by which students take one or more exams to get into the university they want. The university admissions process in different countries is different, but they all have one thing in common: using students' scores when taking the standard exam to apply for university admissions. For example, in the United States, universities do not hold entrance exams. However, they rely on test results from testing organizations independent of educational administration agencies, namely two competitions: the Scholastic Achievement Test and the American College Test [1]. In Japan, 31 specific subjects are selected by the National Education Center to hold the National University Admissions Center Exam [2]. In Vietnam, high school seniors will take a joint exam organized by the Ministry of Education and use those results to apply for university admissions [3]. However, regardless of country, the university entrance exam also has many problems such as cheating, conflict in student choice of university, and university selection of eligible students for admission. Early detection and analysis of these problems can minimize adverse effects and make college admissions more equitable. The College Choice Report [38] reflects college reference of America graduates who took ACT test show that so many students finally did not enroll to any university due to many reasons include they made a wrong decision before. There were 5 ACT Benchmarks Met factors used in this report as Figure 1. The report also show us that the percent of unenrolled students is high, the root of this problem is: (i) can not find a suitable college, or (ii) students did not score good in entrance exam, or (iii) they realized that they made a wrong decision.

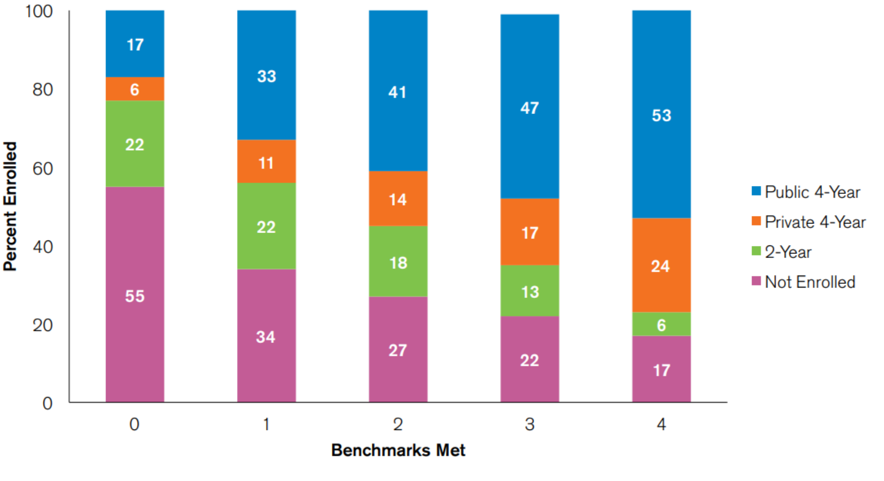

Figure 1. College-type enrollment by number of ACT Benchmarks Met (act.org)

In order to help find a best match between students and universities, Stable matching theory emerged as the best option to solve the problem related to the type of conflict in the matching, and Stable matching theory achieved the Nobel Prize in 2012 (Economic Sciences) [4]. The Stable matching theory is a mathematical framework for analyzing the evolution of cooperative relationships through time in economics. Gale-Shapley algorithms, one of the most used concepts in Stable matching theory, is the foundation of many labor market clearing-houses around the world, and it has been applied to address 1issues with school choice in the USA [5]. In 1962, Gale and Shapley proposed the Gale-Shapley algorithms, a method for solving the stable matching issue. Two cities, Boston and New York, have applied this algorithm in choosing schools [6]. Consequently, the objectives of this research paper include analyzing existing conflicts in admissions that are not adequately resolved. From 1962, when Gale and Shapley published the first study on university admissions, until now, the research results on this type of conflict in university admissions are few. The first publication was by Gale and Shapley in 1962. In this article, an algorithm that the two authors called ”deferred acceptance” laid the foundation for later applications of matching theory. In 1989, Roth also published a publication on this issue. There is rigor among all students, with no indifference or favoritism [7]. However, in this paper, for every pair of stable student outcomes, each college will prefer every student assigned to it in the first outcome but not the second. In a publication by Abdulkadiroglu and Sonmez in 2003, the article pointed out the shortcomings of the problem in several cities ¨ in the USA and proposed two alternative mechanisms [8]. The two authors’ solutions are suitable for all hierarchies and all ages. In June 2003, Abdulkadiroglu also published another publication related to university admissions. Universities will always prefer a group of students who adhere to their major orientations versus a group of students who do not [9]. Additionally, there is competition between private and public institutions, and tuition at public universities is generally less than at private universities due to tax exemptions. In private schools, tuition fees are pretty high, and all students, whether from wealthy or low-income families, have to pay the same tuition fees. This leads students to prefer to go to public universities [10]. In addition, two authors, Balinski and Sonmez, argue that the continuing dictatorship in Turkey causes severe impairments, leads to inefficient admissions, and lowers the quality of education [11].

Hungary has applied Gale-Shapley mechanism in high school and university admissions. It can be said that Hungary is the first country in Europe to apply that method to admissions [12]. Another European country, Germany, has also raised the problem of its admissions when the country’s admission system only prioritizes students with excellent test scores, after which the authors also proposed Gale - Shapley’s algorithm to solve this problem [13, 14, 15]. In Spain, students are not allowed to choose universities but are chosen by the city’s education authority, resulting in colleges not having the opportunity to compete with universities. The authors also proposed Gale - Shapley’s algorithm to solve this [16]. In the Americas, namely Boston - United States, students can choose from the following college choices if they fail the first choice. However, this is risky if those colleges already have enough students [17]. The writers supported the Department of Education in New York City in designing an admissions mechanism based on Gale - Shapley’s algorithm to help more than 90,000 students participate in entrance exams to high school every year [18, 19]. The authors of a recent publication on some issues in charter schools also used Gale-Shapley Algorithms and Boston college admissions data to show that charter schools operate within the school district [20, 21].

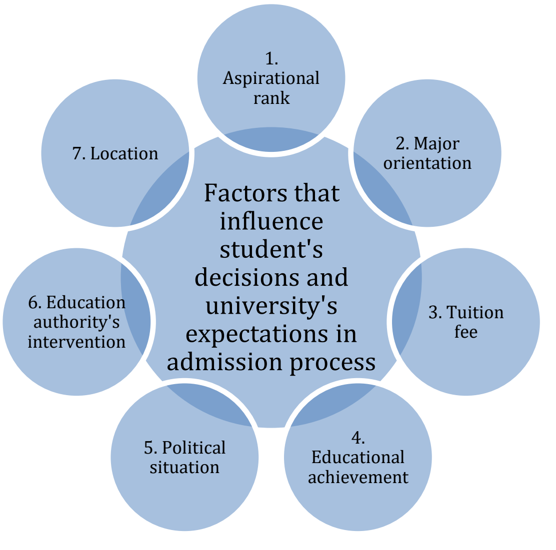

Figure 2. Factors that influence student decisions and university expectations

Most of the above publications have been done for a long time, and many of the issues such publications have not yet been resolved, such as the fact that students prefer urban areas that influence student decisions and university expectations in school choice schools rather than rural schools. As listed above, some researchers used Gale-Shapley algorithm in order to solve the problem of university admision, but there are some isses that we can still improve to fully solve this problem: (i) These related work forcus on analyzed the mathematical model of problem; (ii) The recent Nobel Prize Stable matching theory were not applied in these publication; (iii) There were not many research proposed computational experimence of this problem. Therefore, in this article, we proposed a solutions for Stable Matching for Student Selection (SMSS) using Gale-Shapley algorithm in order to resolve conflicts in university admissions today, including the conflict in student choice of university and selection of university students in admissions, and the algorithm was implemented by an application in Java to bring forward a new approach to this problem.

II. PROBLEM DESCRIPTION

In university admission problems, many high school students are unable to select appropriate institutions due to a lack of strategy, which prevents students from creating an adequate college wish list [31]. There are an array of reasons affecting student's arrangements could be identified. First, the university's reputation is the top concern of students as this will have a significant impact on students when they apply for a job. Employers, who recruit students to work for them, will make some assessment of a student's ability based on the reputation of the school they have attended and this assessment contributes considerably to the student's ability to be recruited or not. Second, there is competition between private and public institutions [10]. Tuition prices in private schools are relatively costly when compared to public school. Whether students come from wealthy or impoverished families, they must pay the same fees. As a result, students opt to attend public universities. This leads to students tending to prefer to go to public universities. Students, third, prefer urban universities rather than rural universities since schools in big cities usually have better quality of education. On the other side, universities always seek out the best students with the highest academic achievement, resulting in many universities failing to meet their enrollment targets. In this paper, we review existing research and put forward new methods to deal with this problem - SMSS. There are three key terms that appear in the research paper: University admission, Stable matching theory, and Gale-Shapley Algorithm. The process of students entering undergraduate studies at universities and colleges is called university admission [30].

A theoretical framework attempting to describe the establishment of mutually beneficial partnerships over time is known as Stable matching theory [31]. A number of labor market clearinghouses throughout the world are built on the Gale-Shapley Algorithm, which has also been applied to the problem of school choice [5]. A match is the term used to describe such a result. In the game theory problem, there is a term used to measure the efficiency of the match, through which the algorithm can be evaluated, called Pareto efficiency, also known as Pareto optimality. A match is said to be Pareto efficient in case there is no other outcome can prove to be better than this outcome for all the players. Specifically in this study, the match is Pareto efficient in case there is no other matching that sends each student to a better school and at least one student are sent to a better school [32].

However, the Gale-Shapley student optimum stable mechanism has certain drawbacks. As an illustration, courtesy of Roth (1982), demonstrates that stability and Pareto efficiency can be mutually exclusive [29].

Example. There are three students i_1, i_2, i_3 and three universities s_1, s_2, s_3 each of which has only one seat. The univerisities' priorities and the students' preferences are as follows:

$$s_1: i_1 - i_3 - i_2 \qquad\qquad i_1: s_2,\ s_1,\ s_3$$

$$s_2: i_2 - i_1 - i_3 \qquad\qquad i_2: s_1,\ s_2,\ s_3$$

$$s_3: i_2 - i_1 - i_3 \qquad\qquad i_3: s_1,\ s_2,\ s_3$$

$$\begin{pmatrix} i_1 & i_2 & i_3 \\ s_1 & s_2 & s_3 \end{pmatrix}$$

Let us interpret the university priorities as university preferences and consider the associated college admissions issue. There is just one stable match in this example:

$$\begin{pmatrix} i_1 & i_2 & i_3 \\ s_2 & s_1 & s_3 \end{pmatrix}$$

For school s_1 and s_2, student i_1 and i_2 have the greatest priority, correspondingly. As a result, student i_1 cannot be sent to a school that is worse than s_1, thus, he or she must be assigned to either s_2 or s_1. Similarly, no school can be allocated to student i_2 that is worse than school s_2, and hence he or she will be assigned to either s_1 or s_2. As a result, students i_1 and i_2 should share schools s_1 and s_2. Stability require them to distribute these schools in an inefficient Pareto manner: This is because if students i_1 and i_2 are allocated to schools s_2 and s_1 approximately, we will have a situation in which student i_3 prefers school s_1 to her assignment and has a greater preference for school s_2 than student i_2 does.

As shown in the Example, completely eliminating justifiable envy may be incompatible with Pareto efficiency. Gale-Shapley student optimum stable mechanism is a particularly mechanism that behaves well if policymakers prioritize total elimination of justifiable envy over Pareto efficiency. For clarity, we modeled the problem as follows: Firstly, all of the students will be sorted into groups of equal size, and the number of groups is equal to the quantity of universities. After clustering, the Stable matching theory will be applied to identify needed characteristics for both parties in SMSS problem. There are some characteristics of players needed taken into account in order to match student and university:

- Type of player: university and student.
- Priority of both student and university when to find the best match, factors can effect a priority of student or university can be: the location of student or university, the reputation of university, grade in entrance exam, etc.
- Preference list of student and preference list of university.

Suppose that there are 8 students *A, B ,C, D , E, F, G, H* applied to 3 universities *X, Y, Z*. All students and universities want to choose the most suitable university and students respectively, which results in competition. To provide more detail for this example, we assigned 8 students randomly to 3 universities then formed pairs *G-Z*. The conflict arises if university *Z* wants to match to other students such as *A, B*, ... and student *G* wants to match to other universities such as *X, Y*. As a result, the matched *G-Z* pair causes a mutual dissatisfaction between student *G* and university *Z*.

Table 1. Before applying matching theory, all pairs are randomly generated

Student	University	Pair
A, B ,C, D , E, F, G, H	*X, Y, Z*	G-Z, A-Y, B-Y, C-Y, D-X, E-X, F-X, G-X, H-X

To balance the wishes of both sides, students make lists of their preferred universities depending on the information supplied about these universities. Universities, similarly, develop a rating of their students preferences based on information provided by students. The Gale-Shapley approach requires these ranked lists of priority as input in the constraints of the algorithm, which is the involvement of 8 students and 3 universities ranking each other. To create preference lists about university, students evaluate universities on several criterion, which is the input of the Gale-Shapley algorithm:

Student's criteria: reputation (the university's position in the university rankings from best to worst), distance (the distance from a student's home to the school), tuition fee (the tuition that students have to pay for school learning), student's university admission rate (based on student's exam and floor scores). Each criteria will be rated on a 5 scale. If there is a criterion = 0, the match will be canceled (that student would not add that school to his/her students' universities selection priority list).

There criterion are also known as the strategy of one player – student in order to choose their university, summary of there criterion are presented in Table 2 as follow.

Table 2. Player's strategy

Player	Player's strategy
Student	Student's entrance exam result Wish list that includes student selection priority, e.g.: 1st school, 2nd school,.., in which school's order is the priority ranking
University	Reputation: the university's position of Distance: the distance between student home and university Tuition fee: of this university Floor score: minimum score that student must achieve to be considered for admission to the university Max: maximum number of student admitted in a year

III. MATHEMATICAL MODEL FOR SMSS

A. *Model of the problem*

The admissions process is complicated progress between students and universities. Because academic ability is not the sole determinant of a student's capability, many universities have tried to add conditions to select the best students for themselves. Instead of applying students based on their first choice, the admissions process is divided into several rounds to widen opportunities for students. For example, each student will create a list of universities they want to attend (in this paper we call it the Preference List) and rank in descending order of importance. Universities are going to base on that order of precedence and their conditions to decide which students will be selected. This principle is applied based on the matching theory, more specifically, stable marriage, which is a problem of coupling a man and a woman so that their pairing is constant [34]. The Deferred - Acceptance Algorithm (DDA), further known as the Gale Shapley algorithm (1962) is the most successful algorithm to solve the problem of stable marriage and be appropriate in university admissions and recruitment processes [7].

Considered an extension of the Gale-Shapley algorithm, the problem of choosing schools between students and universities based on the stable marriage is changed from the original model to achieve the highest matching efficiency [32]. The model based on the main idea is that each student wants to attend a university and they have the university preferences as well as the option of remaining unmatched. Each university wishes to enroll a maximum number of students based on its capacity. Universities have individual ordinal preferences over students and the options to let a student sit unfilled, but they also have responsive preferences over groups of students [33].

Assume that we have two finite and disjoints sets:

$U = \{u_1, .., u_M\}$, in which u_i $(0 < i < M)$ is the i^{th} university that participate in this matching problem, and M is the total number of university.

$S = \{s_1, .., s_N\}$, in which s_j $(0 < j < N)$ is the j^{th} student that participate in this matching problem, and N is the total number of student.

Every university $u_i \in U$ $(0 < i < M)$ has their preference list over the students, and each student $s_j \in S$ $(0 < j < N)$ also has its own preference list over the universities.

Preferences: $PLS_i = \{pls_1, .., pls_{N_i}\}$, is the Preference list of a student i, N_i is the number of suitable university for student i $(0 < i < N)$ and $(0 < N_i < M)$. In which, proposed Preference list of student s_i might be of the form:

$pls_i = \{u_j, u_k, .. u_z\}$ $(0 < i < N)$ and $(0 < j, k, z < M)$.

It means the first choice of the student s_i is the u_j university, the second choice is the u_k university, and so on.

Preferences: $PLU_i = \{plu_1, .., plu_{M_i}\}$, is the Preference list of a university i, M_i is the number of suitable university for student i $(0 < i < M)$ and $(0 < M_i < N)$.

The preference list of a university u_i, for example, , is in a form:

$plu_i = \{s_j, s_k, .. s_z, \emptyset, \emptyset\}$ $(0 < i < M)$ and $(0 < j, k, z < N)$.

This particular list denotes that s_j, s_k, s_z are acceptable students for the university u_i, and there are 2 empty slots left to get students later.

We denote the outcome of the game is the pair-matching $X: U \cup S$, where:

$$\forall s \in S, \exists u \in U, (s, u) \in X$$
$$\forall u \in U, \exists s \in S, (u, s) \in X \quad (1)$$

For any SMSS problem (U, S, P) there always exists at least one stable matching. Because a university can accept many students, so each pair (u, s) is not unique. A matching $x \in X$ called stable if and only if all students who applied to a specific university and every university completes its quota which includes the aggregate amount of students and the quality of entry points. Additionally, the university where the student is admitted must have the highest priority among the schools that are on the student's proposed matriculate list.

To clarify, we will denote $u_i < u_j$ $(0 < i, j < M)$ indicating that a student set that university u_j is higher than u_i in s's preference list. We call the quota of a university is a positive integer $q \in \mathbb{N}$ indicating the highest quantity of positions it required. If we identify a specific university by u_i then its quota shall be indicated by q_i.

To summarize, the model of a SMSS problem is a set (U, S, P), each pair-matching $X: U \cup S$ satisfies the following condition:

$$(\neg\exists(s, u) \notin X) \cap \left(\neg\exists(s, u_i) \in X \cap \left(s, u_j\right) \notin X \cap u_i < u_j\right) \quad (2)$$

B. Gale-Shapley algorithm

As mentioned above, the Gale-Shapley or Deferred Acceptance (DA) algorithm applied in the college admissions problem which was considered an extended two-sided market of the male-female pairing matching. When the DA algorithm is not followed, the output results are unbalanced because some universities do not meet the target, and lots of students are matched to universities they never preferred while the one they wish to enter does not meet the quota. Applied in Boston, USA, the DA algorithm has shown positive results, reducing the number of students entering schools they do not want by 90% [35].

In order to grasp how the Gale-Shapley algorithm works, this section will provide detailed descriptions of this. The algorithm below was proposed based on a study by Diebold et al. in 2014 [36]. The Gale-Shapley algorithm is described through the following steps:

- **Step 0**: All students and universities are free
- **Step 1**: Students sequentially make proposals to each of their most preferred available universities. At any given time, a university can only hold N proposals. Any application received by a university with an open slot will be accepted. Any application from a student whose value is smaller than the existing set of candidates will be rejected by an institution that already has N applications on file.
- **Step k** (k ⩾ 2): If one remains, each student who was turned down in the preceding round makes a proposal for the following selection. These students will be given priority seating at each school according to its allotment. The remaining offers are initially rejected.
- The algorithm stops since all students are matched to universities.

The *Figure 3* below presents the implementation of the Gale-Shapley algorithm applied in SMSS problem:

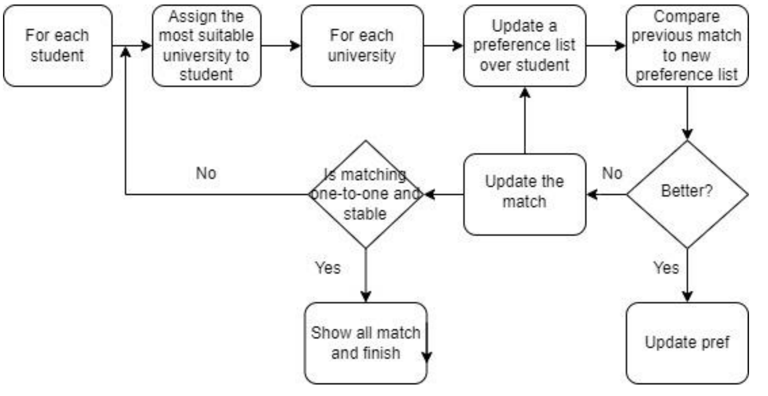

Figure 3. Flowchart of Gale-Shapley alogrithm in SMSS

IV. COMPUTATIONAL EXPERIENCE

In this section, we report the outcome of experiment on SMSS using Gale-Shapley algorithm. The algorithm was coded using Java, and was run on an Ubuntu 22.04.2 LTS, Intel(R) Xeon(R) CPU @ 2.80 GHz., CPU Physical Cores 32, CPU Logical Cores 16, total Memory 251.9 GB.

The data set for the experiments on the algorithm can be summarized in several aspects such as:

Table 3. Preference list of University X = A, B, C, D

Admission Criteria / Student	Reputation (Scale of 5)	Distance (Scale of 5)	Tuition Fee (Scale of 5)	University Admission Rate with Floor Score (Scale of 5)	Total
A	5	4	3	5	17/20
B	5	3	4	4	16/20
C	5	3	4	3	15/20
D	5	2	3	2	12/20
E	5	5	3	0	0/20
F	5	5	3	0	0/20
G	5	1	5	0	0/20
H	5	2	5	0	0/20

Table 4. Preference list of University Y = E, C, A, F, D, B

Admission Criteria / Student	Reputation (Scale of 5) Noted :	Distance (Scale of 5)	Tuition Fee (Scale of 5)	University Admission Rate with Floor Score (Scale of 5)	Total
A	3	2	4	5	14/20
B	3	2	2	5	11/20
C	3	3	4	5	15/20
D	3	1	3	5	12/20
E	3	5	4	5	17/20
F	3	2	3	3	13/20
G	3	1	5	0	0/20
H	3	2	5	0	0/20

When using the Gale-Shaley method, we have an arrangement like the table below. At this step, universities will have 1 more criterion when selecting students, which is maximum number of students admitted (the maximum number of students the university can accept). In case there is only 1 last chance for the last student to be admitted to that school but there are 2 or more students with equal exam scores, the university will select the student based on student's preference list, which students have previously assigned. The highest scores students will be arranged into university first based on the priority that are each other's top favorites. Students with the lower score will be arranged into universities one after another.

Table 5. Preference list of *Student A = X, Y, Z*

Admission Criteria / Universities	Floor Score	Student's test score	Conclusion
X	23	27,5	P
Y	20		P
Z	16		P

Table 6. Preference list of *Student B = X, Y, Z*

Admission Criteria / Universities	Floor Score	Student's test score	Conclusion
X	23	24,5	P
Y	20		P
Z	16		P

In Round 1 of the Gale-Shapley algorithm, the 1st ranked student A is paired 1st with his/her 1st ranked university X at which the student and the university is the best match.

Table 7. Matching in Gale-Shapley in 1 iteration

Round 1: A → X (paired)

Student	University
1. (Score: 27,5) A: X, Y, Z, ø	X (1/2): A, B, C, D, ø
2. (Score: 27,5) B: X, Y, Z, ø	Y (Max: 3): E, C, A, F, D, B, ø
3. (Score: 22) C: Y, Z, ø	Z (Max: 3): E, F, G, H, ø
4. (Score: 21) D: Y, Z, ø E: Y, Z, ø F: Y, Z, ø	
5. (Score: 17) G: Z, ø	
6. (Score: 15) H: ø	

Round 2: B → X (paired)

Student	University
1. (Score: 27,5) A: X, Y, Z, ø	X (Full 2/2): A, B, C, D, ø
2. (Score: 27,5) B: X, Y, Z, ø	Y (Max: 3): E, C, A, F, D, B, ø
3. (Score: 22) C: Y, Z, ø	Z (Max: 3): E, F, G, H, ø
4. (Score: 21) D: Y, Z, ø E: Y, Z, ø F: Y, Z, ø	
5. (Score: 17) G: Z, ø	
6. (Score: 15) H: ø	

Round 7: H → Z (paired)

Student	University
1. (Score: 27,5) A: X, Y, Z, ø	X (Full 2/2): A, B, C, D, ø
2. (Score: 27,5) B: X, Y, Z, ø	Y (Full 3/3): E, C, A, F, D, B, ø
3. (Score: 22) C: Y, Z, ø	Z (Full 2/2): E, F, G, H, ø
4. (Score: 21) D: Y, Z, ø E: Y, Z, ø F: Y, Z, ø	
5. (Score: 17) G: Z, ø	
6. (Score: 15) H: ø	

After solve the problem with sample data above, the student selection result as follow:

University X ↓ A, B	University Y ↓ C, E, F	University Z ↓ D, G, H

We observe that: (i) all students and university have been paired correctly according their data; (ii) the algorithm run pretty fast, the reason may be the short preference list and small number of player of both side (iii) and can not find any other better stable match for all players.

```
function S_UMatching(Universities, Students)
  Matching := List()
  Order := DrawOrder(Students)
  NotPlacable := List()
  while StudentLeftToMatching(Students) do
    MatchingRound(Students, Universities, Order, Matching, NotPlacable)
  end while
  RandomMatching(NotPlacable, Universities, Matching)
  return Matching
end function

function MatchingRound(Students, Universities, Order, Matching, NotPlacable)
  for S ← Students
    List := SelectPreferences(S)
    if UniversityLeftOnList(List) then
      ProposeNextFavorite(List, S, Universities)
    else
      Add(S, NotPlacable)
    end if
  end for
  for U ← Universities do
      Proposals := GetProposals(Universities) - include previous accepted proposals
      SortProposalsToOrderAndPriority(Proposals, Students, Order)
      AcceptMaxNumberOfProposals(Proposals, Students, U, Matching)
  end for
end function
```

Figure 4. Psuedo code of an algorithm

V. CONCLUSION

In this paper, we proposed the problem between universities and students in admission named SMSS. Due to a lack of strategies, many high school students are unable to select good universities and cannot create an adequate college wish list, while a lot of universities fail to reach their enrollment quota because of unsuitable requirements. It is unfair that students who put up their best effort be denied acceptance to the desired school simply because the wish-list is inappropriate and the universities miss the best students that correspond to their level. To solve this conflict, the Stable matching theory, a mathematical model of stable relationships that benefit both parties, was used in order to analyze both players's behaviours and data. The Gale-Shapley algorithm has numerous checking iterations that go over each university on the student's list, resulting in the most equitable output for both the students' and the schools' preferences. This approach bring forward the ability of solving another matching problem sustainably using Stable matching theory and Gale-Shapley algorithm.

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LỰA CHỌN SINH VIÊN CHO TRƯỜNG ĐẠI HỌC: LÝ THUYẾT GHÉP CẶP BỀN VỮNG VÀ GIẢI THUẬT GALE-SHAPLEY

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TÓM TẮT: *Các chiến lược để tăng cường chất lượng giáo dục thường bao gồm việc lựa chọn các phương pháp giáo dục tiên tiến như các kỹ thuật học kết hợp, hoặc học tương tác, tuy nhiên phương thức bền vững hơn để thực hiện việc này cần bắt nguồn từ gốc rễ trong đó người học cần chọn đúng trường theo hoàn cảnh, nhu cầu của mình. Nhiều sinh viên trung học không thể chọn đúng trường học ở bậc tiếp theo của mình bởi vì việc thiếu một chiến lược đúng trong việc ra quyết định lựa chọn, từ đó không thể có được một danh sách lựa chọn các trường phù hợp. Hơn nữa, các trường đại học luôn muốn lựa chọn các sinh viên hàng đầu dựa trên thành tích học tập, từ đó dẫn tới chiến lược tuyển sinh không phù hợp và kết quả là thiếu đi các sinh viên phù hợp nhập học. Để giải quyết vấn đề trên, lý thuyết ghép cặp bền vững cung cấp một mô hình toán học trong việc mô tả các mối quan hệ đôi bên cùng có lợi theo thời gian được áp dụng trong bài viết này. Lý thuyết ứng dụng trong việc ghép các bên bao gồm các sinh viên và các trường đại học, phương pháp thực hiện là coi vấn đề là một trò chơi có tên là Ghép cặp bền vững để lựa chọn sinh viên (SMSS). Liên quan đến bài toán này, thuật toán Gale-Shapley là một phương pháp hữu ích để giải các bài toán ghép nối bền vững, bao gồm nhiều vòng kiểm tra đánh giá sự phù hợp của từng trường đại học trong danh sách sinh viên, ngược lại các trường cũng đánh giá sự phù hợp của sinh viên với trường. Từ những cách giải quyết xung đột này, sinh viên có cơ hội vào học tại những trường đại học phù hợp nhất, giúp họ có thể đóng góp tốt nhất cho xã hội bằng cách có cơ hội học tập bậc cao và tăng cường tri thức. Nhờ đó, họ có được công việc phù hợp, là bước đệm để đóng góp cho xã hội. Bên cạnh đó, giải quyết bài toán sẽ giúp ngăn ngừa sự mất cân đối về số lượng sinh viên giữa các trường đại học, đặc biệt giữa các trường top đầu luôn có quá nhiều đăng ký và ngược lại với các trường kém danh tiếng hơn, từ đó cũng duy trì được nguồn lực ổn định để phát triển các trường này.*