Study on the Control Method for Self-Driving Car Based on Kinematics and Dynamics Mathematics Model

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Abstract—The article describes the analysis of the dynamics and kinematics of a four-wheeled electric vehicle, proposing a controller and evaluating the vehicle's stability during movement. The vehicle's kinematics model is analyzed based on the Ackerman model with 2 front-wheel steering. The inputs to the model consist of parameters related to the vehicle's dimensions. The kinematics analysis showed corresponding results. The vehicle's dynamics are calculated fundamentally, providing inputs for simulating the control and motion problem on the road. To ensure the vehicle operates according to the proposed dynamics and kinematics, accurately providing power to each motor is crucial. Hence, a PID controller is proposed to control the power of each motor precisely. The simulation of the PID controller for the vehicle is conducted using MATLAB-Simulink. The inputs to the simulation include motor parameters, dimensions, vehicle weight, and reference speed. The output is the vehicle's actual speed, which helps evaluate the controller's performance, basis to ensure the feasibility of PID in the experiment with appropriate PID values. Simulating the vehicle running on a trajectory is performed using the AMESIM software. The inputs include vehicle parameters and the reference trajectory. The vehicle is simulated to assess its stability during movement. Obstacles and sensors are also included in the simulation to enable the vehicle to adjust its trajectory when encountering obstacles. The simulation results are the basis for evaluating the safety of the vehicle with various input parameters. This helps in selecting the input parameters that ensure the safety of the vehicle, such as load, speed,... Based on the simulation results, we propose an electric car model with front-wheel steering and use a PID controller to achieve the desired parameters.

Keywords—Four-wheeled electric vehicle, PID controller, autonomous vehicle simulation.

I. INTRODUCTION

Electric vehicles, especially when combined with autonomous driving systems, are at the forefront of current development research. To achieve independent operation, these vehicles require precise perception and control capabilities. Understanding the dynamics and kinematics of the vehicle is vital for controlling its motion, and ensuring accurate trajectory and speed for safety. In our proposed four-wheeled electric vehicle model, we use a PID controller to ensure accurate power delivery to each electric motor. The vehicle model employs the Ackerman geometry with front-wheel steering. Our research focuses on studying the dynamics and kinematics model of this electric vehicle. We simulated the PID controller using MATLAB-Simulink software, considering various parameters related to dimension, weight, motor specifications, power parameters, integrated circuits, and environmental conditions. To evaluate the vehicle's motion and stability, we employed AMESIM software, examining its performance during movement and steering scenarios. Through these simulations, we assessed the controller's stability, accuracy, and the vehicle's operational capabilities, serving as a basis for further advancement and enhancement of autonomous electric vehicle models in real-world applications.

II. KINEMATICS AND DYNAMICS ANALYSIS OF SELF-DRIVING CAR

A. Kinematics model

The power (p) of a vehicle, measured in watts (W), is determined by the relationship between the engine torque (t) and the angular velocity (ω) described by Equation (1).

\[ p = t \cdot \omega \]  \hspace{1cm} (1)

The angular velocity (\( \omega \)) is defined by Equation (2)
The linear velocity \((v)\) is directly proportional to the angular velocity \((\omega)\) and the wheel’s radius \((r)\) is described by Equation (3).
\[
v = \omega \cdot r
\]

The force \((F)\) considers the initial acceleration \((a)\) \((\text{m/s}^2)\) and the vehicle’s mass \((m)\). This relationship is expressed by Equation (4).
\[
F = ma
\]

The motor torque \((t)\) measured in Newton-meters \((\text{N m})\) displacement \((d)\) in meters \((\text{m})\) of the vehicle.

Front-wheel steering (FWS) is the predominant steering model in automotive projects, allowing control of only the front two wheels to alter the vehicle’s direction. It is preferred due to its cost-effectiveness in design and manufacturing, and its ability to reduce the vehicle’s overall weight [1].

\[
\omega = \frac{2\pi}{T} n
\]

Hence, the distance from the center of rotation to the center of the rear wheel gauge, denoted as \((R)\), is determined by Equation 8.
\[
R = \frac{w(\omega_o + \omega_i)}{2(\omega_o - \omega_i)}
\]

From the equation above we obtain the tangents of the internal and external angles. These values are given by Equations (9).
\[
\tan \delta_i = \frac{l}{R - \frac{w}{2}} \quad \text{and} \quad \tan \delta_o = \frac{l}{R + \frac{w}{2}}
\]

**B. Dynamics analysis**

By studying the vehicle and its components, we can determine the forces required to execute specific maneuvers and compensate for various conditions. Establishing a systematic approach to modeling a vehicle’s dynamic system and defining the terminology used to describe its movements is crucial.

**TABLE 1. VARIABLES OF THE KINEMATICS MODEL**

<table>
<thead>
<tr>
<th>Variables</th>
<th>Descriptions</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\delta_i)</td>
<td>Inner wheel steering angle</td>
</tr>
<tr>
<td>(\delta_o)</td>
<td>Outer wheel steering angle</td>
</tr>
<tr>
<td>(w)</td>
<td>Distance between wheels on the same axle</td>
</tr>
<tr>
<td>(l)</td>
<td>Length between the axis</td>
</tr>
<tr>
<td>(R)</td>
<td>Distance from center of rotation to center of distance ((w))</td>
</tr>
<tr>
<td>(\omega_o)</td>
<td>Angular velocity of outer wheels</td>
</tr>
<tr>
<td>(\omega_i)</td>
<td>Angular velocity of internal wheels</td>
</tr>
</tbody>
</table>

The dimensions of many vehicles consist of front and rear gauges that are equal in size, which is crucial for ensuring proper maneuverability. The ideal vehicle model, based on Ackerman’s geometry, is depicted in Figure 1.

The steering angles of the front wheels are determined by the following equations. Equation 7 specifically defines the internal steering angle and the external steering angle.

\[
\delta_i = \operatorname{arctan} \frac{l(\omega_o + \omega_i)}{w \omega_o} \quad \text{and} \quad \delta_o = \operatorname{arctan} \frac{l(\omega_o - \omega_i)}{w \omega_o}
\]

The figure 2 and Table 3 illustrate the conventions for understanding the forces affecting vehicle dynamics. Isaac Newton's second law is commonly used in vehicle dynamics analysis. It applies to translation systems, stating that the total external forces along the \((x)\) axis equal the product of the vehicle's mass and its acceleration in that direction, with the mass considered constant. Equation (12) [2]:
\[
\sum F_x = Ma_x
\]

\[
\sum T_x = I_{xx} a_x
\]

- \(F_x\) - the force of the axis \((x)\).
- \(M\) - the mass of the vehicle.
- \(a_x\) - acceleration on the \((x)\) axis.

In rotation systems, the sum of the torques acting on a body around a specific axis is equal to the product of its moment of rotational inertia and the angular acceleration about the same axis. This relationship is expressed by Equation (13) [2].
\[
\sum T_x = I_{xx} a_x
\]

- \(T_x\) is the motor torque on the axis \((x)\);
- \(I_{xx}\) is the moment of inertia on the \((x)\) axis;
- \(a_x\) is the acceleration on axis \((x)\).

**III. CONTROL DESIGN AND SIMULATION RESULTS FOR SELF-DRIVING CAR**

Simulating the PID controller with a 4-wheel electric vehicle helps ensure feasibility when implementing it in the experiment. By providing actual model parameters as
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inputs, the simulation results have indicated that using a PID controller with appropriate parameters is feasible for the 4-wheeled electric vehicle model. For precise operation, each wheel's motor must receive accurate energy, enabling them to rotate at the intended speed. A PID controller ensures precise power delivery to each motor, simulated using MATLAB-Simulink. Inputs include motor parameters, vehicle dimensions, weight, and reference speed. Output is the actual speed compared to the reference speed after PID adjustments.

![Fig. 3. MATLAB-Simulink result showed the comparison between reference speed and vehicle speed](image)

From the graphs, we can observe that the vehicle's speed in the simulation closely matches the reference speed, indicating that the PID controller's speed control is appropriate for the four-wheeled electric vehicle. The parameters of the PID controller in the simulation are as follows:

\[ K_p = 6.1328; K_d = 4.835; K_i = 0.7435 \]

The four-wheeled electric vehicle uses the Ackerman model, simulated with Amesim software for dynamics and kinematics analysis. First, we create a simulation scenario for a self-driving electric vehicle with the target of following a trajectory along the road. During the movement, obstacles will appear, requiring the vehicle to change its trajectory abruptly. Input parameters, vehicle dimensions, and reference trajectory are considered. Obstacles are included to assess steering ability. Simulation shows successful movement and obstacle avoidance. The AMESIM simulation results indicate that the vehicle operates stably with the specified parameters. When encountering obstacles and requiring significant steering angles, certain parameters like "Car body Lateral acceleration" and "Car body Yaw Velocity" exhibit notable variations. The simulation results help assess the performance of the vehicle when encountering a sharp turn, thereby evaluating how changes in trajectory with a significant amplitude affect the electric car and its sliding capability.

![Fig. 4. Simulation results by AMESIM](image)

By assessing these parameters, we can determine the appropriate load capacity and speed for the vehicle to ensure safety and stability while moving along the trajectory. However, despite these fluctuations, all the parameters eventually return to stable levels as the vehicle continues moving along the initial trajectory. The slip angles of each wheel, being below 3 degrees, ensure that the vehicle's movement remains aligned with the initial trajectory. This observation confirms that the vehicle operates very stably with the input parameters used in the simulation model.

**IV. CONCLUSIONS**

In this paper, we have evaluated the PID controller's capability in controlling the four-wheeled electric vehicle, and we have also assessed the vehicle's stability during movement. Based on these simulation outcomes, we can propose rational design, and control methods for the vehicle model, and these results can help us further enhance to building of the real four-wheeled electric vehicle model.

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