Applying a Modified Differential Evolution for Solving Optimization Problem of Gear Transmission System

Van Tinh Nguyen
School of Mechanical Engineering
Hanoi University of Science and Technology
Hanoi, Vietnam.
tinh.nguyenvan@hust.edu.vn

Abstract—Along with the development of the computer science field, recent decades witness the rise of a class of evolutionary that mimics the process of natural selection to evolve solutions to optimization problems. These algorithms were very successfully applied in solving real-world problems in various fields, such as engineering, economics, and machine learning. The differential evolution (DE) is a well-known algorithm in this group. There are several variants and parameter settings for DE, these parameters need defining to obtain the best configuration of DE for a particular optimization problem. For optimization problem of gear transmission systems, this paper proposed a new mutation operation in which the scaling parameter is multiplied by a random generator that enhances the diversity of the population in each generation. The new strategy was validated with two optimization problems of single stage spur gear and 2-stage planetary gear transmission systems.

Keywords—Differential evolution, mutation operation, random generator, gear transmission.

I. INTRODUCTION

A few recent decades witness a class of many powerful evolutionary algorithms and their applications. Like many other evolutionary algorithms, differential evolution is a population-based technique. This algorithm is developed by Storn and Price [1]. It is inspired by evolution phenomenon in nature and is a strong tool to solve engineering problems. There are three mechanisms (mutation, crossover, and selection) operating in DE in which three control parameters (population size, scaling factor and crossover rate) have significant effect on exploration and exploitation of this algorithm.

Parallel to the development of computer and industry 4.0, evolutionary algorithms gradually demonstrate its pros in comparison with traditionally mathematical method. It results that more and more researchers focus on improvement of evolutionary algorithms and differential evolution has not been lying out this trend.

Literature review on the advance of differential evolution algorithm, we cannot ignore some states of art such as SaDE [2], JDE [3], JADE [4]. Recently, Wang et al. (2014) incorporated the covariance matrix learning and the bimodal distribution parameter setting to generate the control parameters of the mutation and crossover operators. This proposal enhances the capability of DE to solve problems with high variable correlation [5]. Ali Wagdy Mohamed and Ali Khater Mohamed (2019) used two random chosen vectors of the best and worst individuals in each current generation for mutation strategy while the third vector is selected from the middle individuals. This mechanism effectively keeps the balance between the global exploration and local exploitation abilities for searching process [6]. Bui et al. (2019) addressed an improve different evolution algorithm with opposition-based learning mechanism [7].

Gaussian distribution also known as normal distribution is a continuous function which is popularly used in optimization. In the process of the EAs, depending on the principle of each type of various algorithms, Gaussian distribution is applied by the different approaches [8, 9, 10, 11]. Focusing on DE, Lin and Jin randomized scale factor through two Gauss distributions to increase the chance to escape from local optima [12]. Adopting Gaussian mutation is mentioned in [13, 14, 15].

However, there is not a single best algorithm for all optimization problems, each algorithm may work best for one or some specific problems. Thus, this paper applied Gaussian distribution to improve ISADE [16]. The new proposed algorithm is presented, namely G-ISADE. Compared with other evolution algorithms from literature, experimental results indicate that the proposed algorithm performs better than, or at least comparable to state-of-the-art approaches from literature when considering the quality of the solution and convergence speed obtained.

The rest of this paper is organized into three sections. Section 2 describes the proposed method. Section 3 presents the application of the novel approach to gear speed reducer. Finally, Section 4 includes some brief conclusions.
II. MODIFIED DIFFERENTIAL EVOLUTION ALGORITHM

The development of this algorithm is based on an improve self-adaptive differential evolution algorithm proposed by Bui et al. (2013) [16]. It mainly consists of three operations.

A. Mutation strategy with Gaussian distribution

Gaussian distributions play an important role in statistics and are often used to generate real-valued random variables. Gaussian distribution gives a fantastic possibility to control the exploiting zone in optimization based on complexity of each considered problem. This strategy generates a new trial vector by multiply Gaussian random variable to a vector difference in mutation mechanism of DE as described in (1) and (2).

\[ DE/best/1: U_{i,j}^{iter} = X_{best,j} + N(\mu, \sigma^2) \times F^*(X_{iter}^{r_1,j} - X_{iter}^{r_2,j}) \] (1)

\[ DE/best/2: U_{i,j}^{iter} = X_{best,j} + N(\mu, \sigma^2) \times F^*(X_{iter}^{r_1,j} - X_{iter}^{r_2,j}) + F^*(X_{iter}^{r_3,j} - X_{iter}^{r_4,j}) \] (2)

where: \( V \), mutation vector; \( X \), current vector; \( X_{best} \), best fitness of current vector; \( iter \), number of iterations; \( i \), index of number of particles in population; \( NP \); \( j \), index of the number of dimensions; \( D \); \( r_1, r_2, r_3, \) and \( r_4 \) are different elements chosen randomly from \([1; NP]\); and \( NP, D \), number of populations and dimensions, respectively. \( N(\mu, \sigma^2) \) is a Gaussian random variable with mean \( \mu \) and standard deviation \( \sigma \). Recommended values for \( \mu \) and \( \sigma \) are 1 and 0.1, respectively.

Scaling factor \( F \) is set to be high in the first iteration and after certain generations, it becomes smaller for proper exploitation. The value \( F \) is generated by a Sigmoid function as in (3).

\[ F_i = \frac{1}{1 + \exp(-a \frac{j - NP \times I}{NP})} \] (3)

where: \( a \) is the parameter that controls the value of scaling factor \( F \).

B. Crossover operation

After mutation operation, DE adopts a crossover operation as (4).

\[ U_{i,j}^{iter} = \begin{cases} V_{i,j}^{iter}, & \text{if } (\text{rand}[0,1]) \leq CR \text{ or } j = j_{\text{rand}} \\ X_{i,j}^{iter}, & \text{otherwise} \end{cases} \] (4)

where: \( U, V \), and \( X \) are a trial vector, mutation vector, and current vector, respectively. CR is a crossover parameter within the interval \((0, 1)\). \( j_{\text{rand}} \) is an integer random number selected from set \([1, 2, ..., D]\).

C. Selection operation

This stage will choose the better fitness between trial vector \( U \) and target vector \( X \) for the next generation as (5).

\[ X_{i}^{iter+1} = \begin{cases} U_{i}^{iter}, & \text{if } f(U_{i}^{iter}) \leq f(X_{i}^{iter}) \\ X_{i}^{iter}, & \text{otherwise} \end{cases} \] (5)

III. MODIFIED DIFFERENTIAL EVOLUTION FOR OPTIMIZATION PROBLEM OF GEAR REDUCER

Gear reducer is widely applied in machinery, it is used to reduce the speed and increase the torque taken by the motor. This research applied the modified differential evolution to two optimization problems: Single stage and 2-Stage planetary spur gear reducer.

A. Single stage spur gear reducer

This problem is described in [17] and show in Fig. 1, it contains seven design variable as follow: the face width, \( x_1 \); the gear module, \( x_2 \); the number of teeth on pinion, \( x_3 \); the length of the first shaft, \( x_4 \); the length of the second shaft, \( x_5 \); diameter of the first shaft, \( x_6 \); and the diameter of the second shaft, \( x_7 \). Objective function is to minimize the weight of speed reducer expressed in Equation 6. This problem is bounded by eleven constraint functions as described in (6) to (17).

Design variables:

\( x_1 \in [2.6, 3.6]; x_2 \in [0.7, 0.8]; x_3 \in [17, 28], x_3 \) is an integer;

\( x_4 \in [7.3, 8.3]; x_5 \in [7.3, 8.3]; x_6 \in [2.9, 3.9], x_7 \in [5, 5.5] \)

Minimize

\[ f(x) = 0.7854x_1x_2^3(3.3333x_3^3 + 14.9334x_3 - 43.0934) - 1.508x_1(x_5^2 + x_6^2) + 7.4777(x_6^2 + x_7^2) + 0.7854(x_4x_6^2 + x_6x_7^2) \] (6)

Subject to

\( g_1 = 27.0x_4^{-1}x_5^2x_6^{-1} - 1.0 \leq 0 \) (7)

\( g_2 = 397.5x_4^{-1}x_5x_6^{-1} - 1.0 \leq 0 \) (8)

\( g_3 = 1.93x_4^2x_5^2x_6^{-4} - 1.0 \leq 0 \) (9)

\( g_4 = 1.93x_5^2/x_4x_6x_7^2 \leq 1.0 \leq 0 \) (10)

\( g_5 = 1.0/(110x_6^2) \sqrt((745x_4^2/x_2x_3)^2 + 16.9 \times 10^6) - 1.0 \leq 0 \) (11)

\( g_6 = 1.0/(85x_7^2) \sqrt((745x_5^2/x_2x_3)^2 + 157.5 \times 10^6) - 1.0 \leq 0 \) (12)

\( g_7 = x_3x_4/40.0 - 1.0 \leq 0 \) (13)

\( g_8 = 5.0x_4/x_1 - 1.0 \leq 0 \) (14)

\( g_9 = x_4/(12.0x_3) - 1.0 \leq 0 \) (15)

\( g_{10} = (1.5x_4 + 1.9)/x_3 - 1.0 \leq 0 \) (16)

\( g_{11} = (1.1x_4 + 1.9)/x_3 - 1.0 \leq 0 \) (17)
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Table I shows the comparison of the results with previous research, we define an error (e) which is maximum of constraint functions with optimal solution, Max(g(x, opt)).

**TABLE I. COMPARISON OF THE RESULTS WITH OTHER ALGORITHMS.**

<table>
<thead>
<tr>
<th>Method</th>
<th>Optimal solution</th>
<th>Objective value</th>
<th>e</th>
</tr>
</thead>
<tbody>
<tr>
<td>Akhtar et al. [18]</td>
<td>(3.506122, 0.700006, 17, 7.549126, 7.859330, 3.365576, 5.289773)</td>
<td>3008.197440</td>
<td>&lt;10⁻⁶</td>
</tr>
<tr>
<td>Tosserams et al. [19]</td>
<td>(3.5, 0.7, 17, 7.3, 7.72, 3.35, 5.29)</td>
<td>2996.645783</td>
<td>&lt;10⁻³</td>
</tr>
<tr>
<td>Lin et al. [20]</td>
<td>(3.50, 0.7, 17, 7.3, 7.715, 3.349, 5.286)</td>
<td>2993.73892</td>
<td>&lt;10⁻²</td>
</tr>
<tr>
<td>Huang [21]</td>
<td>(3.50, 0.7, 17, 7.3, 7.715, 3.350282, 5.286654)</td>
<td>2990.124384</td>
<td>&lt;10⁻²</td>
</tr>
<tr>
<td>Li and Papalambros [22]</td>
<td>(3.500000, 0.7, 17, 7.500000, 7.800000, 3.350214, 5.286863)</td>
<td>2994.471921</td>
<td>&lt;10⁻⁶</td>
</tr>
<tr>
<td>ISADE [16]</td>
<td>(3.5, 0.7, 17.0, 7.3)</td>
<td>2996.348165</td>
<td>0 (-1.1E⁻²)</td>
</tr>
<tr>
<td>G-ISADE</td>
<td>(3.5, 0.7, 17.0, 7.3, 7.713886, 3.350215, 5.285351)</td>
<td>2993.611549</td>
<td>16</td>
</tr>
</tbody>
</table>

Fig. 2. Convergence speed of optimization process on single stage spur gear reducer problem.

**B. 2-Stage planetary gear reducer**

Planetary gear reducers play an important role in transmission systems and are commonly applied in industry. It has many pros in comparison with normal transmission systems such as light weight, compact size, large speed ratio, high efficiency.

This section considers a 2-Stage planetary gear reducer as described in Fig. 3. This problem involves nice design variables: the numbers of teeth of the first stage sun gear and the second stage sun gear (x₁, x₂), the gear module (x₅, x₆), the face width (x₇, x₈), the modification coefficient of the sun gear (x₉, x₁₀), the transmission ratio of the first stage (x₇). The target is a minimum of weight as displayed in (18) under ten constraint functions which include stress limitations, adjacent condition expressed in (19) to (28). The details of problem can be found in [23].

![Fig. 3. 2-Stage planetary gear reducer.](image)

Subject to

\[
g₁ = 2x₂8.995x6.9x1.5x₃ + ((x₅x₆(x₈x₉)²) - (303.41/(2.22x189.98x0.95))² ≤ 0 \quad (19)
\]

\[
g₂ = 2x₂8.95x1.78x10⁶ × (30x₃x₄ + 1)/(x₅x₆x₇x₈) × 30/x₉ - (1104.47/(2.25x189.98x0.94))² ≤ 0 \quad (20)
\]

\[
g₃ = 2x₂8.95x6.9x10⁵/(x₅x₆x₇x₈) × 30/x₉ - (499.39/(2.29x1.73x1.12)) ≤ 0 \quad (21)
\]

\[
g₄ = 2x₂8.995x1.78x10⁶/x₅x₆x₇x₈ - (521.53/(2.32x1.73x1.08)) ≤ 0 \quad (22)
\]

\[
g₅ = x₅x₆x₇x₈x₉x₁₀ - 1.3x₈x₉x₁₀ ≤ 0 \quad (23)
\]

\[
g₆ = 0.6x₅x₆x₇x₈x₉x₁₀ ≤ 0 \quad (24)
\]

\[
g₇ = 0.6x₅x₆x₇x₈x₉x₁₀ ≤ 0 \quad (25)
\]

The result of optimization is obtained as follow: x₁ = 20; x₂ = 4.957651; x₃ = 59.557864; x₄ = 0.5; x₅ = 5.423887; x₆ = 24, x₇ = 4.907367, x₈ = 147.951034, x₉ = 0.5.

![Fig. 4. Convergence speed of optimization process on 2-Stage planetary gear reducer problem.](image)
IV. CONCLUSIONS

This paper addressed the application of the new approach for solving optimization problem of gear speed reducer. The proposed algorithm is an improved version of ISADE, it adopts a Gaussian random variable for mutation operation. The new method was successfully applied to solve two optimization problems of single stage spur gear and 2-stage planetary gear transmission systems. It is confirmed that the proposed method significantly improved the convergence speed and accuracy in comparison with the previous approaches. By introducing G-ISADE algorithm, this research contributes one more option to engineers when solving the real problems in their work.

REFERENCES