Dynamic Behavior of a Functionally Graded Beam Under a Moving Load on Nonlinear Viscoelastic Foundation Considering Moving Mass

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Abstract
In this paper, linear dynamic analysis of a functionally graded materials (FGMs) beams on nonlinear viscoelastic foundation under to moving load considering moving mass has been investigated. The nonlinear behavior of viscoelastic foundation is represented by the third order relationship. The governing equation of motion of the beams is derived based on Hamilton principle expressed as Lagrange’s equations with specific boundary conditions satisfied with Lagrange’s multiplier and sporadic by high order polynomial. The computer program using Newmark-β time integration and Newton Raphson procedure is written by MATLAB language. Material properties of the beam vary continuously in thickness direction according to a power law form. The effects of the material distribution, velocity of the moving load, mass of load, parameters of foundation as linear, shear and nonlinear layer on the displacement of the beam have been examined in detail.

Keywords: Beams, Functionally graded beams, Nonlinear foundation, Moving mass, Dynamic analysis.

1. Introduction

Today, the development of computer science and the increasing demand of people requires materials with high durability, application. Therefore, scientists who Japan proposed multiple layers of composite material that is functionally graded material. energy in 1980s. In terms of mechanics, multi-layer composite material has the advantages over base materials to create it because they are distributed in the appropriate position with the features of each type of material. FGMs are special composites that have a continuous variation of material properties from one surface to another. These new kinds of material have been employed for a wide range of applications such as defense industries, aircrafts, space vehicles, etc. Studies devoted to understand the static and dynamic behavior of the FGM beams on the ground under dynamic loads is one of the problems attracting many researchers around the world. This problem is highly practical, to describe behavior for many structures such as roads, railways, airport landing surfaces. To survey behavior for many on the above structure, we need to select the model of beams and ground soil to suit the structural analysis problem.

Based on the above discussions, there have been several investigated on free vibration analysis of functionally graded beams. Free vibration analysis of FG beams based on Timoshenko theory and Hamiltonian principle, examined the effects of different boundary conditions, material distribution coefficient and oscillation mode to conduct FGMs beams [1]. Nonlinear dynamic analysis of FGMs beams subjected to moving harmonic loads based on Timoshenko theory combining deformation and nonlinear relationship Von-Kerman and equations of motion of the system obtained from the equation Lagranges, vertical and horizontal displacements are approximated by polynomial forms [2]. Vibration analysis of FGMs beams in different theory subjected to moving mass and the effect of shear deformation, the inertia, Coriolis, the moving mass on the dynamic displacements and the tresses of the beam [3]. Research Euler beam under to moving harmonic load by the finite element method, influence of the coefficient foundation and frequency of excitation force on the beams are investigated [4]. The dynamic behavior of simply-supported isotropic beams subjected to an eccentric axial load and a moving harmonic load was analyzed by using Euler–Bernoulli, Timoshenko and the third order shear deformation theory [5], [6]. Fundamental frequency analysis of FG beams by different beam theories [7]. The investigation of the effects of inertial, centripetal, and Coriolis forces on the dynamic response of a cracked beam under moving mass load [8]. Dynamic response of a simply supported beam subjected to moving masses is considered [9].

More recently, there have been few investigated on effects of different parameters of elastic foundation on behavior of FG beams [10], [11]. Investigation of the dynamic stability of (FGSW) and (FGO) by FEM on elastic foundation [12]. Dynamic responses of beam subjected to moving load and moving mass supported by Pasternak foundation based on Euler-Bernoulli [13],[14]. Along with the development of computer science, humankind has proposed many models with different foundation are more parameters recommended for accurate results, close to reality [15]. The multi-parameter foundation model was created to describe nonlinear behavior in the process of working with the above structure.
Recently, a number of third-order nonlinear foundation models proposed the relationship between force and displacement in the structural dynamics problem on the ground, giving results close to reality. Confirmed through comparative research results turn deformed rails foundation model of linear, nonlinear and experimental foundation fact [16]. In addition to analyzing nonlinear foundation behavior, it makes more realistic view, to describe more realistically. Most recently, analyzed the dynamic behavior of beams on the nonlinear foundation under load [17,18]. The stability analysis of beam cross section changes based nonlinear elastic foundation subjected to moving load by FEM, the results indicate the influence of the boundary conditions of the beam with parameters different foundation to the vertical displacement of the beam [19]. Studying dynamic behavior of FGM beams based on nonlinear elastic substrates including shearing layer and cubic nonlinearity and effect of elastic foundation coefficients associated with boundary conditions and nonlinear behavior [20]. Study nonlinear behavior of FG beams on nonlinear elastic foundation under to axial force based on Euler-Bernoulli [21]. The nonlinear equations of motion are solved via Newmark–Euler–Bernoulli method [22]. Through this, the foundation model is being researchers worldwide interest and also suggested in recent times. Dynamic analysis of beam on a new foundation model nonlinear behavior under to a moving mass [23]. However, the behavior of the beams on the nonlinear viscoelastic foundation under a moving load considering moving mass has not been studied much, especially the application materials in the form of structural FGMs has not mentioned much. The aim of this paper is to investigate behavior of FGM beams on nonlinear viscoelastic foundation under a moving load. Newmark and Newton Raphson method is applied to the equations of motion of the beams. Also, examine the influence of parameters such as the effects of the material distribution, velocity of the moving load, parameters of foundation as linear, shear, nonlinear layer and mass load on the dynamic responses of the beam are discussed in detail.

2. Functionally graded materials

We consider a FGMs: L, b, h, with coordinate system (Oxyz) having the origin O and respectively, resting on an nonlinear viscoelastic foundation under to a moving load considering moving mass is show in Fig. 1.
Dynamic behavior of a functionally graded beam under a moving load on nonlinear viscoelastic foundation

Considering Moving Mass

Fig. 1. A FGMs beams on nonlinear foundation due to moving mass M.

It is assumed that the FGMs beams is made of ceramic and metal. According to the rule of mixture, the effective material properties [2]:

\[
P(z) = (P_{c} - P_{m}) \left( z/h + 0.5 \right)^k + P_{m}
\]  

(1)

Based on the third order shear deformation theory, the axial displacement, \( u(x, z, t) \), and the transverse displacement of the beam, \( w(x, z, t) \) are:

\[
u(x, z, t) = u_{0}(x, t) - z \frac{\partial u_{0}(x, t)}{\partial x} + \Phi(z) \left( \frac{\partial u_{0}(x, t)}{\partial x} - \phi_{0}(x, t) \right)
\]

\[
w(x, z, t) = \omega_{0}(x, t)
\]

(2a)

(2b)

Timoshenko beam theory (FSDBT): \( \Phi(z) = z \)

(3)

The strain-displacement relations are given by:

\[
\varepsilon_{axx} = \frac{\partial u}{\partial x} - z \frac{\partial^{2} w}{\partial x^{2}} + \Phi(z) \left( \frac{\partial^{2} u}{\partial x^{2}} - \frac{\partial \phi}{\partial x} \right)
\]

\[
\gamma_{a} = \frac{\partial w}{\partial x} + \frac{\partial^{2} u}{\partial x \partial z} + \frac{\partial \Phi}{\partial x} \left( \frac{\partial u}{\partial x} - \phi \right)
\]

(4a)

(4b)

By assuming that the material of FGMs beams obeys Hooke’s law, the stresses in the beam become:

\[
\sigma_{axx} = E(z) \varepsilon_{axx}
\]

\[
\tau_{a} = G(z) \gamma_{a}
\]

(5a)

(5b)

By neglecting the rotary inertia of the moving mass, the kinetic energy of the moving mass is

\[
K_{m} = \frac{1}{2} M \left[ \left( \frac{\partial u_{0}}{\partial t} + v_{m} \right)^2 + \left( \frac{\partial w_{0}}{\partial t} + v_{m} \frac{\partial \phi_{0}}{\partial t} \right)^2 \right]_{t=t_{0}}
\]

(6)

The strain energy of the beam is

\[
U = \frac{1}{2} \int_{L} \left[ \sigma_{a} \varepsilon_{axx} + \tau_{a} \gamma_{a} \right] dA dx
\]

(7)

\[
U = \frac{1}{2} \int_{L} \left( A_{m} \left( \frac{\partial u_{0}}{\partial t} \right)^2 + 2(E_{m} - B_{m}) \frac{\partial u_{0}}{\partial t} \frac{\partial w_{0}}{\partial t} - 2E_{m} \frac{\partial \phi_{0}}{\partial t} \frac{\partial \phi_{0}}{\partial t} \right) dA dx
\]

\[
+ \left( D_{m} + H_{m} - 2F_{m} \right) \left( \frac{\partial^{2} u_{0}}{\partial x^{2}} \right)^2 + 2(F_{m} - H_{m}) \left( \frac{\partial^{2} w_{0}}{\partial x^{2}} \right)^2 + \frac{A_{m}}{2} \left( \frac{\partial \psi_{0}}{\partial t} \right)^2 dA dx
\]

\[
+ \frac{1}{2} \int \left[ (\Phi(z))^{2} \right] G(z) dA
\]

(8)

where

\[
(E_{m}, F_{m}, D_{m}) = \int E(z)(1, z, z^{2}) dA
\]

\[
(H_{m}) = \int \Phi(z)(z) E(z) dA
\]

\[
(A_{m}) = \int \Phi(z) E(z) dA
\]

(9)

The deformation energy in the nonlinear foundation is

\[
U_{f} = \frac{1}{2} k_{l} w^{2} dx + \frac{1}{2} \int_{L} k_{f} \left( \frac{\partial w}{\partial x} \right)^2 dx + \frac{1}{4} \int_{L} k_{nl} w^{4} dx
\]

(10)

The kinetic energy of the beam at any instant is

\[
K = \frac{1}{2} \int \rho(z)(v_{1}^{2} + v_{2}^{2}) dA dx
\]

\[
\left[ \frac{\partial^{2} u_{0}}{\partial t^{2}} \right] + 2(I_{k} - I_{h}) \left( \frac{\partial u_{0}}{\partial t} \right) \frac{\partial u_{0}}{\partial t} \left( \frac{\partial \omega_{0}}{\partial x} \right) \frac{\partial \omega_{0}}{\partial x}
\]

\[
+ 2\left( I_{k} - I_{h} \right) \left( \frac{\partial \phi_{0}}{\partial t} \right) \frac{\partial \omega_{0}}{\partial x} + I_{h} \left( \frac{\partial \phi_{0}}{\partial t} \right) \frac{\partial \phi_{0}}{\partial t}
\]

\[
- 2I_{k} \frac{\partial u_{0}}{\partial t} \frac{\partial \phi_{0}}{\partial t} + I_{h} \left( \frac{\partial \omega_{0}}{\partial t} \right)^2
\]

(11)

By defining the following cross-sectional inertial coefficients:

\[
(I_{x}, I_{y}, I_{p}) = \int \rho(z)(1, z, z^{2}) dA
\]

\[
(I_{x}, I_{y}) = \int \Phi(z)(1, z) dA
\]

\[
(I_{p}) = \int \Phi(z)^{2} \rho(z) dA
\]

(13a)

(13b)

(13c)

Potential of the moving load at any instant [3]:

\[
V = -M_{g} \omega_{0}(x_{m}, t) \left[ c(t - t_{1}) - c(t - t_{2}) \right]
\]

(14)

The Lagrange multipliers formulation of the considered problem requires constructing the Lagrangian functional as follows:
The equations of motion (20) written as compact:

\[ [M + MS][\ddot{q}(t)] + [C + 2Mv_em]\dot{q}(t) + [K]\{q(t)\} = \{P_m(t)\} \]

where \( \{q(t)\} = \{A(t), B(t), C(t), \alpha(t)\}^T \) are the generalized coordinates. The following frequency equation can be expressed in the following matrix form:

\[ [K][\ddot{\sigma}] - \omega^2[M][\ddot{\sigma}] = 0 \]

The following non-dimensional frequency [15].

\[ \lambda = \omega L^2 \sqrt{\frac{I_a}{K_1[k^2/\rho^2]}} \quad ; \quad I_1 = \int^{h/2} \rho(x) dx \]

### 3. Simulation results

In the numerical results, dynamic responses of a FGMs beams on nonlinear viscoelastic foundation due to moving loads has been investigated.

#### Table 1: Material property of FGMs constituents.

<table>
<thead>
<tr>
<th>Property</th>
<th>Unit</th>
<th>Aluminum</th>
<th>Zirconia</th>
</tr>
</thead>
<tbody>
<tr>
<td>E (GPa)</td>
<td></td>
<td>70</td>
<td>200</td>
</tr>
<tr>
<td>( \rho ) (kg/m³)</td>
<td></td>
<td>2700</td>
<td>5700</td>
</tr>
<tr>
<td>( \nu )</td>
<td></td>
<td>-0.3</td>
<td>0.3</td>
</tr>
</tbody>
</table>

#### Table 2: The dimensions and attribute of the beam.

<table>
<thead>
<tr>
<th>RL (m)</th>
<th>b (m)</th>
<th>h (m)</th>
<th>k₁</th>
<th>k₂</th>
<th>k₃</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>0.5</td>
<td>0.5</td>
<td>5/6</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

#### Table 3: The characteristic of foundation

<table>
<thead>
<tr>
<th>RL</th>
<th>M₁</th>
<th>M₂</th>
<th>M₃</th>
<th>K₁</th>
<th>K₂</th>
<th>K₃</th>
<th>( \xi )%</th>
</tr>
</thead>
<tbody>
<tr>
<td>250</td>
<td>5</td>
<td>10</td>
<td>5</td>
<td>5</td>
<td>20</td>
<td>5</td>
<td>10</td>
</tr>
</tbody>
</table>

#### Table 4: Properties of load

<table>
<thead>
<tr>
<th>P(kN)</th>
<th>( m_r )</th>
<th>( v(m/s) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1500</td>
<td>1</td>
<td>20</td>
</tr>
</tbody>
</table>

#### Table 5: Examined the number of the time steps (RL)

<table>
<thead>
<tr>
<th>RL</th>
<th>W_max (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>250</td>
<td>0.0687</td>
</tr>
<tr>
<td>500</td>
<td>0.0686</td>
</tr>
<tr>
<td>750</td>
<td>0.0685</td>
</tr>
<tr>
<td>1000</td>
<td>0.0685</td>
</tr>
</tbody>
</table>
subsequent calculations (see table 5). The numerical results are compared with the previous works to demonstrate the performance of the present study. Firstly, to further verify the present results, natural frequencies of FGMs beams composed of alumina and aluminum are calculated and compared with before study for k = 0.3 and L/h = (10, 30, 100) in table 6. The following material and beam properties: h = 0.5m. 
Alumina: $E_{c} = 380$ GPa, $\rho_{c} = 3800$ kg/m$^{3}$, $v_{c} = 0.23$ 
Aluminum: $E_{c}=70$GPa, $\rho_{c} = 2700$kg/m$^{3}$, $v_{c} = 0.23$

Table 6. Comparison of non-dimensional fundamental frequencies.

<table>
<thead>
<tr>
<th>BC</th>
<th>Author</th>
<th>L/h = 10</th>
<th>L/h = 30</th>
<th>L/h = 100</th>
</tr>
</thead>
<tbody>
<tr>
<td>S-S</td>
<td>[7]</td>
<td>2.701</td>
<td>2.738</td>
<td>2.742</td>
</tr>
<tr>
<td></td>
<td>[6]</td>
<td>2.702</td>
<td>2.738</td>
<td>2.742</td>
</tr>
<tr>
<td></td>
<td>Article</td>
<td>2.774</td>
<td>2.813</td>
<td>2.817</td>
</tr>
<tr>
<td>C-F</td>
<td>[7]</td>
<td>0.970</td>
<td>0.976</td>
<td>0.977</td>
</tr>
<tr>
<td></td>
<td>[6]</td>
<td>0.996</td>
<td>1.003</td>
<td>1.003</td>
</tr>
<tr>
<td></td>
<td>Article</td>
<td>0.970</td>
<td>0.976</td>
<td>0.977</td>
</tr>
<tr>
<td>C-C</td>
<td>[7]</td>
<td>5.875</td>
<td>6.177</td>
<td>6.214</td>
</tr>
<tr>
<td></td>
<td>Article</td>
<td>5.811</td>
<td>6.167</td>
<td>6.212</td>
</tr>
</tbody>
</table>

The comparisons show that the agreement between the present results and those of [6], [7] is satisfactory.

The dynamic deflections under to moving load with time t are presented by the following property of beam, load and foundation in [17]. Chart of the beam displacement over time (see Fig. 2) relatively consistent with the solution [17]. The article applies Timshenko beam theory with the generalized coordinate method while the result [17] applies Euler-Bernoulli beam theory with the Galerkin and Runge-Kutta methods. So the displacement line in 1/3 of the time at the beginning and 1/3 of the last time, there is a difference but not significant and it is acceptable. Relatively small displacement value less dangerous for the structure.

Fig. 2. The vertical displacements of beam (k = 0) Survey beams (S-S) boundary conditions with the following parameters for k = 0, L = b = 1m, h = [0.1; 0.2; 0.5] m, E = 206 (GPa), $\rho = 7860$ (kg/m$^{3}$) are calculated and compared with those of [12], [14].

Table 7: Comparison of fundamental nondimensional frequency.

<table>
<thead>
<tr>
<th>L/h</th>
<th>$K_{1}$</th>
<th>$K_{2}$</th>
<th>Article</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[14]</td>
<td>[12]</td>
<td>FSDBT</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
<td>7.412</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0</td>
<td>8.010</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>0</td>
<td>12.11</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>1</td>
<td>12.01</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>1</td>
<td>12.38</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>1</td>
<td>15.32</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>0</td>
<td>9.27</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0</td>
<td>9.78</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>0</td>
<td>13.54</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>1</td>
<td>13.45</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>1</td>
<td>13.80</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>1</td>
<td>16.68</td>
</tr>
</tbody>
</table>
The comparison is provided in Table 7. It is found that the present results are in good agreement.

The dynamic deflections subjected to moving mass are presented by the following formula in [9]

\[ s_{ubj} = \frac{m_r}{t^2} \]

The vertical displacements under the moving mass for full metal beam with time \( t \).

The comparisons show that the present dynamic displacements are in good agreement with the result [9] for \( m_r = 0.1 \) see Fig. 3.

Fig. 4. When the foundation coefficient is increased, the stiffness of the foundation increases, leading to a stiffening of the large beam so that the displacement of the beam decreases.

Fig. 5. When the material distribution coefficient \( k \) increases, the material moves from ceramic to metal. The ceramic has a large hardness, but the metal has a small hardness, so the displacement of the beam increases.

The larger the volume load is proportional to the displacement of the beam, consistent physical properties see Fig. 6.

The larger the mass ratio, the more the displacement of the beam will increase. Besides, in the case of beam displacements under a moving mass larger than the case of beam moving load see Fig. 7.
4. Conclusions

Dynamic responses of a functionally graded materials beams on nonlinear viscoelastic foundation due to moving loads considering moving mass has been investigated. The effects of the material distribution, velocity of the moving mass, parameters of foundation as linear, shear, viscoelastic and nonlinear layer on the displacement of the beams have been examined. From the results analyzed above the following conclusions are reached:

- When linking two ends of the beam as hard, the oscillation frequency of the beam increases.
- The material distribution coefficient of the beam increases, the metal content in the beam is greater than ceramic whose metal has a smaller hardness than ceramic resulting in the most soft structure and maximum displacement.
- The harder the ground, the higher the structure also increases in stiffness, so the beam’s displacement is significantly reduced.

References


